4.2

Congruence and Triangles

What you should learn

GOAL 1 Identify congruent figures and corresponding parts.

GOAL 2 Prove that two triangles are congruent.

Why you should learn it

▼ To identify and describe congruent figures in **real-life** objects, such as the sculpture described in **Example 1**.

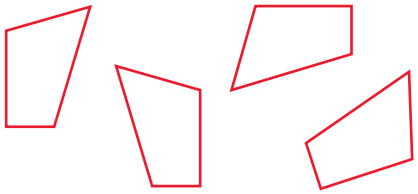
REAL LIFE

Two Open Triangles Up Gyratory II by George Rickey

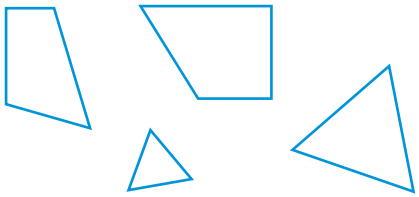
GOAL 1 IDENTIFYING CONGRUENT FIGURES

Two geometric figures are *congruent* if they have exactly the same size and shape. Each of the red figures is congruent to the other red figures. None of the blue figures is congruent to another blue figure.

Congruent

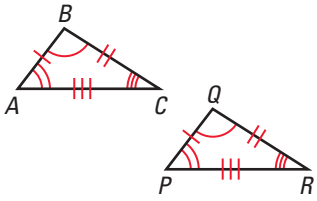


Not congruent



When two figures are **congruent**, there is a correspondence between their angles and sides such that **corresponding angles** are congruent and **corresponding sides** are congruent. For the triangles below, you can write $\triangle ABC \cong \triangle PQR$, which is read "triangle ABC is congruent to triangle PQR." The notation shows the congruence and the correspondence.

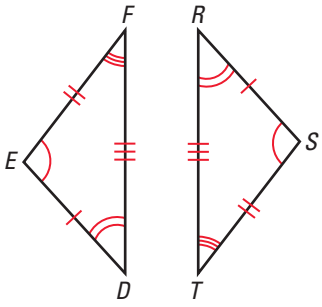
Corresponding angles	Corresponding sides
$\angle A \cong \angle P$	$\overline{AB} \cong \overline{PQ}$
$\angle B \cong \angle Q$	$\overline{BC} \cong \overline{QR}$
$\angle C \cong \angle R$	$\overline{CA} \cong \overline{RP}$



There is more than one way to write a congruence statement, but it is important to list the corresponding angles in the same order. For example, you can also write $\triangle BCA \cong \triangle QRP$.

EXAMPLE 1 Naming Congruent Parts

The congruent triangles represent the triangles in the photo above. Write a congruence statement. Identify all pairs of congruent corresponding parts.



SOLUTION

The diagram indicates that $\triangle DEF \cong \triangle RST$. The congruent angles and sides are as follows.

Angles: $\angle D \cong \angle R$, $\angle E \cong \angle S$, $\angle F \cong \angle T$

Sides: $\overline{DE} \cong \overline{RS}$, $\overline{EF} \cong \overline{ST}$, $\overline{FD} \cong \overline{TR}$

STUDENT HELP

Study Tip

Notice that single, double, and triple arcs are used to show congruent angles.

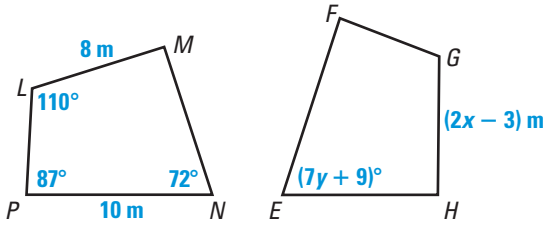
Using Algebra

EXAMPLE 2 Using Properties of Congruent Figures

In the diagram, $NPLM \cong EFGH$.

a. Find the value of x.

b. Find the value of y.



SOLUTION

a. You know that $\overline{LM} \cong \overline{GH}$.
So, $LM = GH$.

$8 = 2x - 3$

$11 = 2x$

$5.5 = x$

b. You know that $\angle N \cong \angle E$.
So, $m\angle N = m\angle E$.

$72° = (7y + 9)°$

$63 = 7y$

$9 = y$

. . . . . .

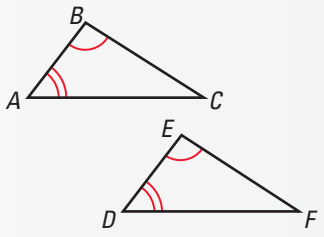
The Third Angles Theorem below follows from the Triangle Sum Theorem. You are asked to prove the Third Angles Theorem in Exercise 35.

THEOREM

THEOREM 4.3 Third Angles Theorem

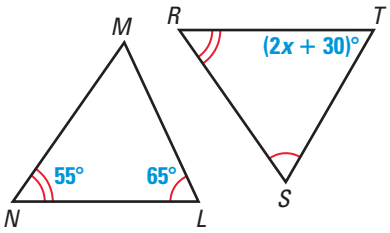
If two angles of one triangle are congruent to two angles of another triangle, then the third angles are also congruent.

If $\angle A \cong \angle D$ and $\angle B \cong \angle E$, then $\angle C \cong \angle F$.



EXAMPLE 3 Using the Third Angles Theorem

Find the value of x.



SOLUTION

In the diagram, $\angle N \cong \angle R$ and $\angle L \cong \angle S$. From the Third Angles Theorem, you know that $\angle M \cong \angle T$. So, $m\angle M = m\angle T$. From the Triangle Sum Theorem, $m\angle M = 180° - 55° - 65° = 60°$.

$m\angle M = m\angle T$ — Third Angles Theorem

$60° = (2x + 30)°$ — Substitute.

$30 = 2x$ — Subtract 30 from each side.

$15 = x$ — Divide each side by 2.

STUDENT HELP

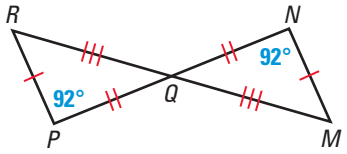
INTERNET HOMEWORK HELP

Visit our Web site www.mcdougallittell.com for extra examples.

GOAL 2 PROVING TRIANGLES ARE CONGRUENT

EXAMPLE 4 Determining Whether Triangles are Congruent

Decide whether the triangles are congruent. Justify your reasoning.



Proof

SOLUTION

Paragraph Proof From the diagram, you are given that all three pairs of corresponding sides are congruent.

$\overline{RP} \cong \overline{MN}$, $\overline{PQ} \cong \overline{NQ}$, and $\overline{QR} \cong \overline{QM}$

Because $\angle P$ and $\angle N$ have the same measure, $\angle P \cong \angle N$. By the Vertical Angles Theorem, you know that $\angle PQR \cong \angle NQM$. By the Third Angles Theorem, $\angle R \cong \angle M$.

So, all three pairs of corresponding sides and all three pairs of corresponding angles are congruent. By the definition of congruent triangles, $\triangle PQR \cong \triangle NQM$.

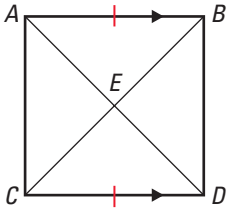
EXAMPLE 5 Proving Two Triangles are Congruent

FOCUS ON APPLICATIONS

REAL LIFE **TRIANGULAR STAMP**

When these stamps were issued in 1997, Postmaster General Marvin Runyon said, "Since 1847, when the first U.S. postage stamps were issued, stamps have been rectangular in shape. We want the American public to know stamps aren't 'square.'"

The diagram represents the triangular stamps shown in the photo. Prove that $\triangle AEB \cong \triangle DEC$.



GIVEN ▶ $\overline{AB} \parallel \overline{DC}$, $\overline{AB} \cong \overline{DC}$, E is the midpoint of $\overline{BC}$ and $\overline{AD}$.

PROVE ▶ $\triangle AEB \cong \triangle DEC$

Plan for Proof Use the fact that $\angle AEB$ and $\angle DEC$ are vertical angles to show that those angles are congruent. Use the fact that $\overline{BC}$ intersects parallel segments $\overline{AB}$ and $\overline{DC}$ to identify other pairs of angles that are congruent.

SOLUTION

Statements	Reasons
1. $\overline{AB} \parallel \overline{DC}$, $\overline{AB} \cong \overline{DC}$	1. Given
2. $\angle EAB \cong \angle EDC$, $\angle ABE \cong \angle DCE$	2. Alternate Interior Angles Theorem
3. $\angle AEB \cong \angle DEC$	3. Vertical Angles Theorem
4. E is the midpoint of $\overline{AD}$, E is the midpoint of $\overline{BC}$.	4. Given
5. $\overline{AE} \cong \overline{DE}$, $\overline{BE} \cong \overline{CE}$	5. Definition of midpoint
6. $\triangle AEB \cong \triangle DEC$	6. Definition of congruent triangles

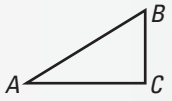
In this lesson, you have learned to prove that two triangles are congruent by the *definition of congruence*—that is, by showing that all pairs of corresponding angles and corresponding sides are congruent. In upcoming lessons, you will learn more efficient ways of proving that triangles are congruent. The properties below will be useful in such proofs.

THEOREM

THEOREM 4.4 ***Properties of Congruent Triangles***

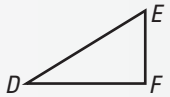
REFLEXIVE PROPERTY OF CONGRUENT TRIANGLES

Every triangle is congruent to itself.



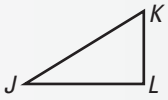
SYMMETRIC PROPERTY OF CONGRUENT TRIANGLES

If $\triangle ABC \cong \triangle DEF$, then $\triangle DEF \cong \triangle ABC$.



TRANSITIVE PROPERTY OF CONGRUENT TRIANGLES

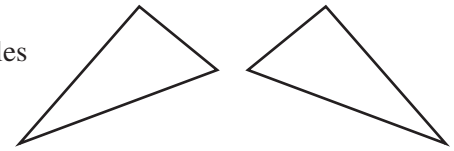
If $\triangle ABC \cong \triangle DEF$ and $\triangle DEF \cong \triangle JKL$, then $\triangle ABC \cong \triangle JKL$.



GUIDED PRACTICE

Vocabulary Check ✓

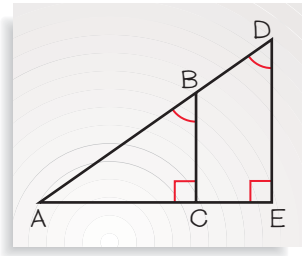
1. Copy the congruent triangles shown at the right. Then label the vertices of your triangles so that $\triangle JKL \cong \triangle RST$. Identify all pairs of congruent *corresponding angles* and *corresponding sides*.



Concept Check ✓

ERROR ANALYSIS **Use the information and the diagram below.**

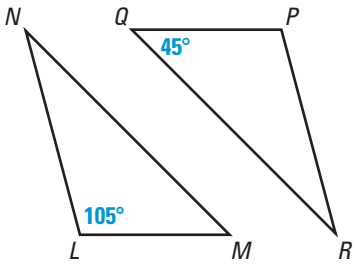
On an exam, a student says that $\triangle ABC \cong \triangle ADE$ because the corresponding angles of the triangles are congruent.



2. How does the student know that the corresponding angles are congruent?
3. Is $\triangle ABC \cong \triangle ADE$? Explain your answer.

Skill Check ✓

Use the diagram at the right, where $\triangle LMN \cong \triangle PQR$.



4. What is the measure of $\angle P$?
5. What is the measure of $\angle M$?
6. What is the measure of $\angle R$?
7. What is the measure of $\angle N$?
8. Which side is congruent to $\overline{QR}$?
9. Which side is congruent to $\overline{LN}$?

PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice to help you master skills is on p. 809.

DESCRIBING CONGRUENT TRIANGLES **In the diagram, $\triangle ABC \cong \triangle TUV$. Complete the statement.**

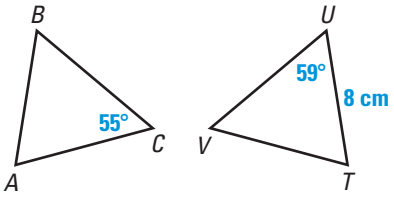
10. $\angle A \cong$?

11. $\overline{VT} \cong$?

12. $\triangle VTU \cong$?

13. $BC =$?

14. $m\angle A = m\angle$? $=$? $^\circ$



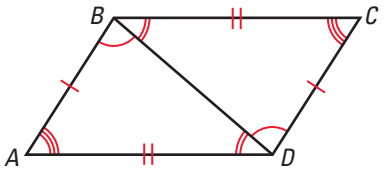
15. Which of the statements below can be used to describe the congruent triangles in Exercises 10–14? (There may be more than one answer.)

A. $\triangle CBA \cong \triangle TUV$ **B.** $\triangle CBA \cong \triangle VUT$

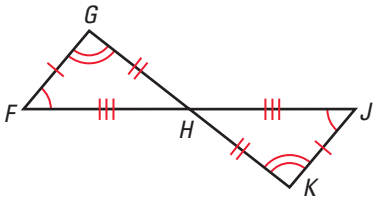
C. $\triangle UTV \cong \triangle BAC$ **D.** $\triangle TVU \cong \triangle ACB$

NAMING CONGRUENT FIGURES **Identify any figures that can be proved congruent. Explain your reasoning. For those that can be proved congruent, write a congruence statement.**

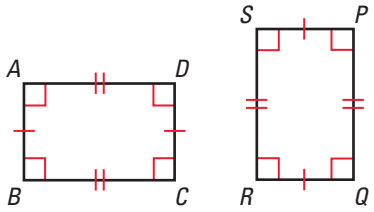
16.



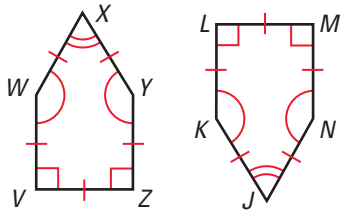
17.



18.



19.



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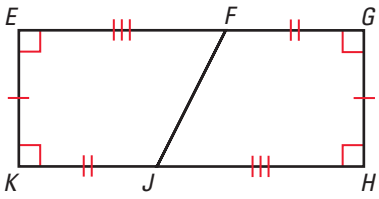

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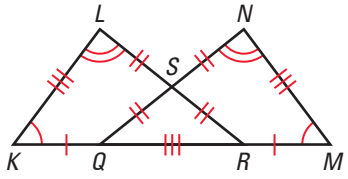


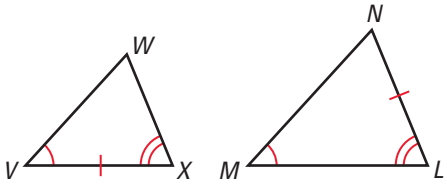
STUDENT HELP

HOMEWORK HELP

Example 1: Exs. 10–22
Example 2: Exs. 14, 24, 25
Example 3: Exs. 26–29
Example 4: Exs. 16–21, 23
Example 5: Ex. 38

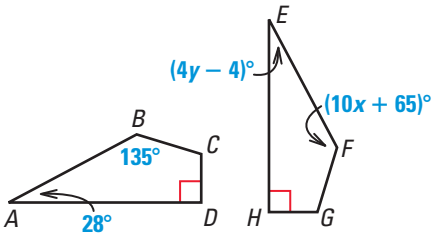
22. IDENTIFYING CORRESPONDING PARTS Use the triangles shown in Exercise 17 above. Identify all pairs of congruent corresponding angles and corresponding sides.

23. CRITICAL THINKING Use the triangles shown at the right. How many pairs of angles are congruent? Are the triangles congruent? Explain your reasoning.

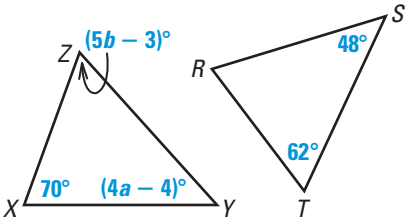


USING ALGEBRA **Use the given information to find the indicated values.**

24. Given $ABCD \cong EFGH$, find the values of x and y.

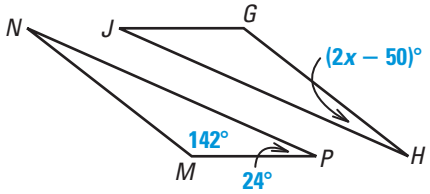


25. Given $\triangle XYZ \cong \triangle RST$, find the values of a and b.

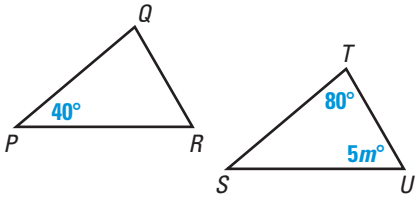


USING ALGEBRA **Use the given information to find the indicated value.**

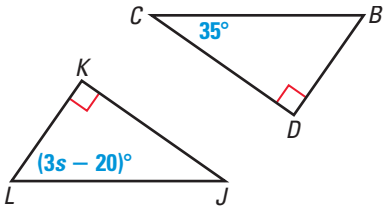
26. Given $\angle M \cong \angle G$ and $\angle N \cong \angle H$, find the value of x.



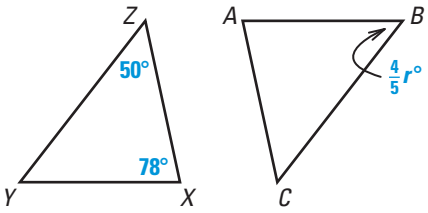
27. Given $\angle P \cong \angle S$ and $\angle Q \cong \angle T$, find the value of m.



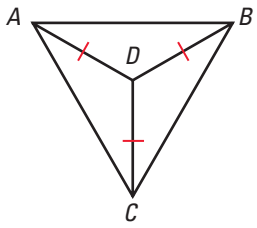
28. Given $\angle K \cong \angle D$ and $\angle J \cong \angle C$, find the value of s.



29. Given $\angle A \cong \angle X$ and $\angle C \cong \angle Z$, find the value of r.



CROP CIRCLES **Use the diagram based on the photo. The small triangles, $\triangle ADB$, $\triangle CDA$, and $\triangle CDB$, are congruent.**



This pattern was made by mowing a field in England.

30. Explain why $\triangle ABC$ is equilateral.

31. The sum of the measures of $\angle ADB$, $\angle CDA$, and $\angle CDB$ is 360°. Find $m\angle BDC$.

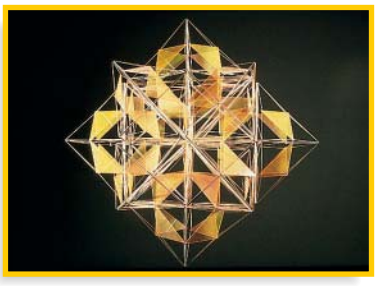
32. Each of the small isosceles triangles has two congruent acute angles. Find $m\angle DBC$ and $m\angle DCB$.

33. **LOGICAL REASONING** Explain why $\triangle ABC$ is equiangular.

FOCUS ON PEOPLE

HARRIET BRISSON is an artist who has created many works of art that rely on or express mathematical principles. The pattern used to arrange the triangles in her sculpture shown at the right can be extended indefinitely.

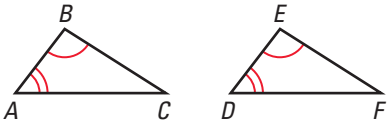
34. SCULPTURE The sculpture shown in the photo is made of congruent triangles cut from transparent plastic. Suppose you use one triangle as a pattern to cut all the other triangles. Which property guarantees that all the triangles are congruent to each other?



35. DEVELOPING PROOF Complete the proof of the Third Angles Theorem.

GIVEN ▶ $\angle A \cong \angle D, \angle B \cong \angle E$

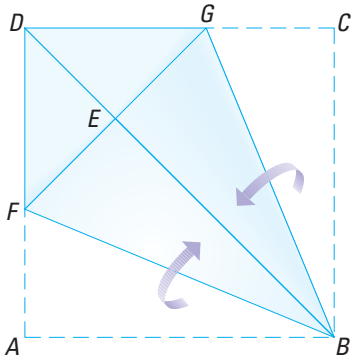
PROVE ▶ $\angle C \cong \angle F$



Statements	Reasons
1. $\angle A \cong \angle D, \angle B \cong \angle E$	1. ?
2. $m\angle$? $= m\angle$?, $m\angle$? $= m\angle$?	2. ?
3. $m\angle A + m\angle B + m\angle C = 180°$, $m\angle D + m\angle E + m\angle F = 180°$	3. ?
4. $m\angle A + m\angle B + m\angle C = m\angle D + m\angle E + m\angle F$	4. ?
5. $m\angle D + m\angle E + m\angle C = m\angle D + m\angle E + m\angle F$	5. ?
6. $m\angle C = m\angle F$	6. ?
7. ?	7. Def. of $\cong \measuredangle$s.

ORIGAMI **Origami is the art of folding paper into interesting shapes. Follow the directions below to create a kite. Use your kite in Exercises 36–38.**

1. Fold a square piece of paper in half diagonally to create $\overline{DB}$.
2. Next fold the paper so that side $\overline{AB}$ lies directly on $\overline{DB}$.
3. Then fold the paper so that side $\overline{CB}$ lies directly on $\overline{DB}$.



36. Is $\overline{EB}$ congruent to $\overline{AB}$? Is $\overline{EF}$ congruent to $\overline{AF}$? Explain.

37. LOGICAL REASONING From folding, you know that $\overrightarrow{BF}$ bisects $\angle EBA$ and $\overrightarrow{FB}$ bisects $\angle AFE$. Given these facts and your answers to Exercise 36, which triangles can you conclude are congruent? Explain.

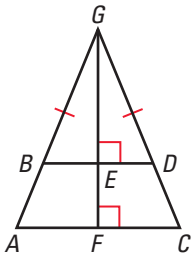
38. PROOF Write a proof.

GIVEN ▶ $\overline{DB} \perp \overline{FG}$, E is the midpoint of $\overline{FG}$, $\overline{BF} \cong \overline{BG}$, and $\overrightarrow{BD}$ bisects $\angle GBF$.

PROVE ▶ $\triangle FEB \cong \triangle GEB$

Test Preparation

39. MULTI-STEP PROBLEM Use the diagram, in which $ABEF \cong CDEF$.



a. Explain how you know that $\overline{BE} \cong \overline{DE}$.

b. Explain how you know that $\angle ABE \cong \angle CDE$.

c. Explain how you know that $\angle GBE \cong \angle GDE$.

d. Explain how you know that $\angle GEB \cong \angle GED$.

e. *Writing* Do you have enough information to prove that $\triangle BEG \cong \triangle DEG$? Explain.

★ Challenge

40. ORIGAMI REVISITED Look back at Exercises 36–38 on page 208. Suppose the following statements are also true about the diagram.

$\overrightarrow{BD}$ bisects $\angle ABC$ and $\overrightarrow{DB}$ bisects $\angle ADC$.
$\angle ABC$ and $\angle ADC$ are right angles.

EXTRA CHALLENGE
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Find all of the unknown angle measures in the figure. Use a sketch to show your answers.

MIXED REVIEW

DISTANCE FORMULA **Find the distance between each pair of points.** (Review 1.3 for 4.3)

41. $A(3, 8)$, $B(-1, -4)$

42. $C(3, -8)$, $D(-13, 7)$

43. $E(-2, -6)$, $F(3, -5)$

44. $G(0, 5)$, $H(-5, 2)$

45. $J(0, -4)$, $K(9, 2)$

46. $L(7, -2)$, $M(0, 9)$

FINDING THE MIDPOINT **Find the coordinates of the midpoint of a segment with the given endpoints.** (Review 1.5)

47. $N(-1, 5)$, $P(-3, -9)$

48. $Q(5, 7)$, $R(-1, 4)$

49. $S(-6, -2)$, $T(8, 2)$

50. $U(0, -7)$, $V(-6, 4)$

51. $W(12, 0)$, $Z(8, 6)$

52. $A(-5, -7)$, $B(0, 4)$

FINDING COMPLEMENTARY ANGLES **In Exercises 53–55, $\angle 1$ and $\angle 2$ are complementary. Find $m\angle 2$.** (Review 1.6)

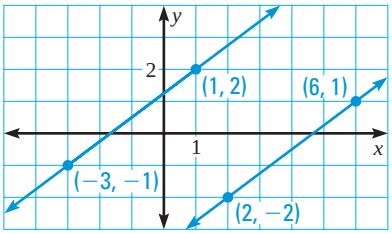
53. $m\angle 1 = 8°$, $m\angle 2 =$?

54. $m\angle 1 = 73°$, $m\angle 2 =$?

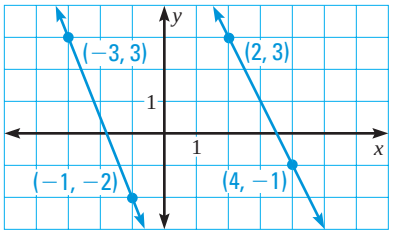
55. $m\angle 1 = 62°$, $m\angle 2 =$?

IDENTIFYING PARALLELS **Find the slope of each line. Are the lines parallel?** (Review 3.6)

56.



57.

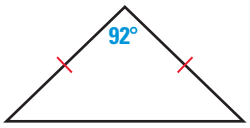


QUIZ 1

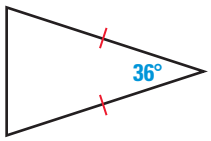
Self-Test for Lessons 4.1 and 4.2

Classify the triangle by its angles and by its sides. (Lesson 4.1)

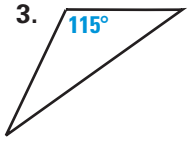
1.



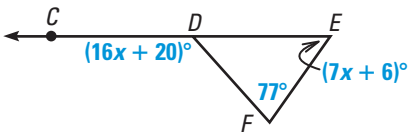
2.



3.



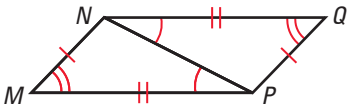
4. Find the value of x in the figure at the right. Then give the measure of each interior angle and the measure of the exterior angle shown. (Lesson 4.1)



Use the diagram at the right. (Lesson 4.2)

5. Write a congruence statement. Identify all pairs of congruent corresponding parts.

6. You are given that $m\angle NMP = 46°$ and $m\angle PNQ = 27°$. Find $m\angle MNP$.



MATH & History — Triangles In Architecture

APPLICATION LINK
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THEN

AROUND 2600 B.C., construction of the Great Pyramid of Khufu began. It took the ancient Egyptians about 30 years to transform 6.5 million tons of stone into a pyramid with a square base and four congruent triangular faces.

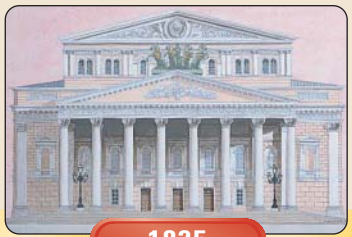
NOW

TODAY, triangles are still used in architecture. They are even being used in structures designed to house astronauts on long-term space missions.

1. The original side lengths of a triangular face on the Great Pyramid of Khufu were about 219 meters, 230 meters, and 219 meters. The measure of one of the interior angles was about 63°. The other two interior angles were congruent. Find the measures of the other angles. Then classify the triangle by its angles and sides.

c. 2600 B.C.

Construction on the Great Pyramid of Khufu begins.



1825

Moscow's Bolshoi Theater uses triangles in its design.

1990s

Architect Constance Adams uses triangles in the design of a space module.